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FINITE-RANGE DWBA ANALYSIS OF (p, α) REACTIONS ON ^{9}Be AND ^{11}B

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Abstract: The ground state angular distributions of the reactions ^{9}Be(p, α)^{6}Li and ^{11}B(p, α)^{8}Be are analyzed by means of the DWBA theory. One-step direct mechanisms only are considered, but for both reactions the inclusion of the heavy-particle pick-up is found to be essential. DWBA fits to the data are obtained (a) in the zero-range approximation, (b) in the "fixed-range" approximation and (c) by an exact finite-range calculation. Only the latter method yields both an acceptable fit and reasonable values for the spectroscopic factors.

1. Introduction

The DWBA is a very successful tool in the analysis of direct nuclear reactions. For single- or double-nucleon transfer it usually permits the extraction from the data of the quantum numbers and the spectroscopic parentage of the nuclear states directly involved. In recent years this theory has also been applied with increasing frequency to multi-nucleon exchange reactions, although the basic assumptions under which such calculations are performed still stand on a rather doubtful basis. One of the most questionable features of such a procedure is the use of the zero-range approximation. With the advent of fast computers with large memories, however, this approximation can now be dispensed with, as is done in the present paper. The results of the calculation then become a test of the quality of the cluster model of the nuclear structures involved and of the assumption that the nucleons participating in the transfer are exchanged, tightly bound together, in one step. Since, as will be seen below, the question as to which mechanisms contribute significantly to the reaction is not *a priori* solved, the quality of the fits is to a certain extent an indication of the correctness of assumptions made in this respect.

In the present paper, DWBA is used to obtain fits to the ground state angular distributions of the reactions ^{9}Be(p, α)^{6}Li and ^{11}B(p, α)^{8}Be, both measured [1]) at an incident prot on energy of 45 MeV. Such processes are usually analyzed under the assumption that they entirely proceed by means of a triton pick-up. The fact, however, that the cross sections under consideration also exhibit strong backward peaking, besides the forward peak predicted by the triton-transfer hypothesis, suggests that another mechanism, the

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heavy-particle pick-up, must be of considerable importance. Hence, the calculations presented here include the contributions of ^{5}He pick-up and ^{7}Li pick-up for the targets ^{9}Be and ^{11}B, respectively. The question as to whether the transfer of so many nucleons is adequately described by a cluster model of the reaction is left open. The reasonable fits and spectroscopic factors obtained perhaps suggest that this simplified picture at least forms a useful basis for better approximations, such as a two-particle $(\alpha+n)$ transfer description of the ^{5}He exchange, for instance.

The choice of the optical-model potentials used to describe the scattering states is explained in sect. 2. The problem there consists in the choice of the parameters for the heavy-ion channel: all values were obtained from interpolation or extrapolation of quantities determined by fits to available elastic scattering data of α-particles on lithium and beryllium. All potentials used in the computation of the DWBA cross sections were then treated as fixed quantities and not adjusted for the purpose of securing better fits.

The DWBA analysis is presented in sect. 3. First, zero-range results are shown, including the contributions of the triton and of the heavy-particle pick-ups. Because the zero-range approximation is particularly questionable in cases where the bound state on which the δ-function is applied is not a 1s state, the heavy-particle pick-ups (for which this reservation applies) are also evaluated in the "fixed-range" approximation. Finally, the results of an exact finite-range DWBA calculation are presented and their meaning is discussed.

TABLE 1

Proton optical-model parameters

	V	r_0	a	W_V	W_S	r'_0	a'	$V_{s.o.}$
$p+{}^9$Be	66.90	0.801	0.710	4.69	0.00	2.049	0.629	2.95
$p+{}^{11}$B	51.17	1.110	0.570	7.50	8.08	1.110	0.500	5.50

Definition:

$$U = -V(e^x+1)^{-1} - i(W_V - 4W_S \frac{d}{dx'}(e^{x'}+1)^{-1} + \frac{h}{m_\pi c^2} V_{s.o.}\, \boldsymbol{\sigma}\cdot\boldsymbol{L}\, r^{-1}\left(\frac{d}{dr}\right)(e^x+1)^{-1},$$

$$x = \frac{r - r_0 A^{\frac{1}{3}}}{a},$$

$$x' = \frac{r - r'_0 A^{\frac{1}{3}}}{a'}.$$

The Coulomb potential from a uniformly charged sphere of radius 1.2 $A^{\frac{1}{3}}$ fm is added. Strengths are in MeV, lengths in fm.

2. The optical-model parameters

2.1. PROTON PARAMETERS

The optical-model parameters for the scattering of 45 MeV protons by ^{9}Be used in the present calculations were obtained by Satchler [2]) from fits to data measured at

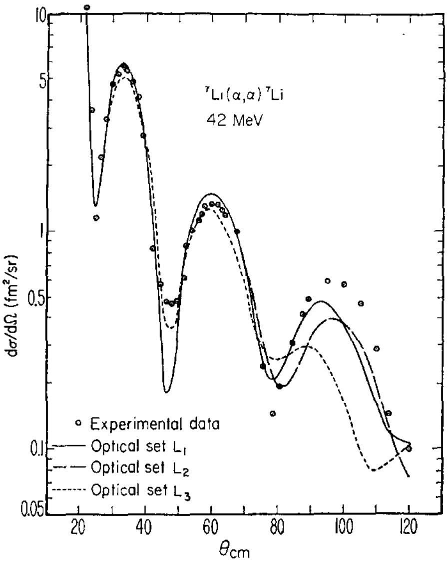

Fig. 1. Optical-model fits to the elastic scattering of 42 MeV α-particles on ^7Li. The data are from ref. [6]; the parameters are listed in table 2.

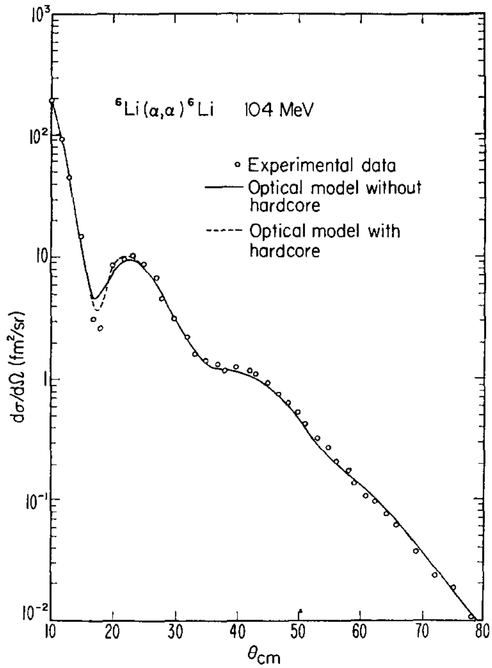

Fig. 2. Optical-model fits to the elastic scattering of 104 MeV α-particles on ^6Li. The data and the fit with hard core are from ref. [7]; the parameters are listed in table 2.

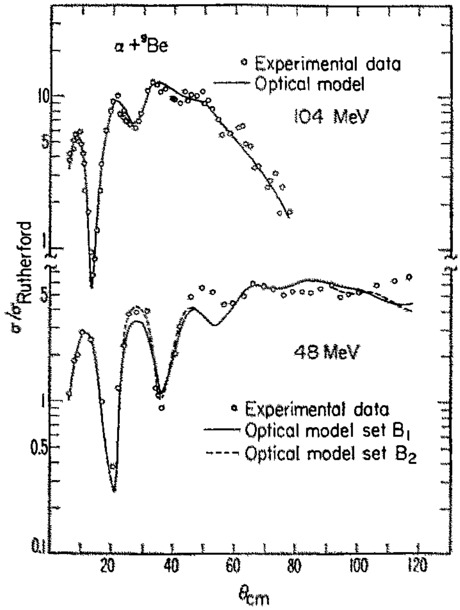

Fig. 3. Optical-model fits to the elastic scattering of 104 and 48 MeV α-particles on ^{9}Be. The data are from refs. [7,8]); the parameters are listed in table 3.

TABLE 2

Final-state (α+Li) optical parameters

	V	r_0	a	W_V	W_S	r'	a'	χ^2
$\alpha+{}^7$Li 42 MeV								
set L1	98.34	0.948	0.660	2.17	4.61	2.365	0.383	102
set L2	122.43	0.768	0.756		7.39	2.125	0.496	127
set L3	53.13	1.586	0.444	5.44		2.412	0.576	193
$\alpha+{}^6$Li 104 MeV	88.86	0.991	0.807	4.94	0.00	3.006	0.577	133
average (with set L1)	94.6	0.969	0.733	3.05	2.30	2.685	0.480	

See table 1 for definition of the potentials.

TABLE 3

Final-state ($\alpha+{}^9$Be) optical parameters

	V	r	r	W_V	W_S	r'	a'	χ^2
$\alpha+{}^9$Be 48 MeV								
set B1	71.77	1.544	0.691	50.78		0.953	0.918	433
set B2	67.15	1.669	0.653		22.44	1.066	0.720	361
$\alpha+{}^9$Be 104 MeV	65.87	1.483	0.655	34.94		1.057	1.054	342
average BA1	68.8	1.514	0.673	42.9		1.005	0.986	
BA2	66.5	1.576	0.654	25.39	11.22	1.062	0.887	

See table 1 for definition of potentials.

UCLA [3]). Only his best set is chosen here and the values are listed in table 1.

The parameters for the scattering of protons on ^{11}B were determined by means of a prescription suggested by Watson *et al.* [4]), which appears to yield good fits to the elastic-scattering data of protons on many light nuclei in a broad band of energies. Again the values are listed in table 1.

2.2. ALPHA PARAMETERS

Fits were performed by means of the optical-model search code SEEK [ref. [5])] on available α-particle scattering data on ^{6}Li, ^{7}Li and ^{9}Be at various energies to determine the potentials for the DWBA calculations.

Specifically, in the case of the reaction $^{9}Be(p, \alpha)^{6}Li$ at $E_p = 45$ MeV, one needs parameters for the scattering of 75 MeV α-particles on ^{6}Li. The available data consist of scattering from [6]) ^{7}Li at 42 MeV and from [7]) ^{6}Li at 104 MeV. The optical-model fits obtained here are presented in figs. 1 and 2 and the potentials thus determined are listed in table 2, together with the average set (an average of set L1 of the 42 MeV data and of the 104 MeV set) used for the DWBA computations.

In the case of the reaction $^{11}B(p, \alpha)^{8}Be$, elastic scattering data of α-particles on ^{9}Be at [8]) 48 MeV and at [7]) 104 MeV were used. The optical-model fits are shown in fig. 3 and their parameters listed in table 3. Since both sets, B1 and B2, seem to equally well describe the experimental points, two average sets, called BA1 and BA2 in table 3, were extracted and used in the DWBA calculations with very similar results, as will be shown in sect. 3.

Other ambiguities in the search for optical-model parameters were also encountered. A detailed description of the procedures used to obtain the present parameters can be found elsewhere [9]).

As mentioned in sect. 1, the parameters determined here were not subsequently adjusted during the calculation of the DWBA cross sections.

3. DWBA analysis

3.1. ZERO-RANGE PREDICTIONS

The zero-range DWBA calculations were performed by means of the computer code DWUCK [ref. [10])], slightly modified to permit the simultaneous evaluation of the contributions of several mechanisms and their sum. Spin-orbit forces were taken into account here for the incident channel. Several sets of values of the total (j) and orbital (l) angular-momentum transfers, as well as of the spin transfer (s), had to be taken into account, as prescribed by the selection rules discussed in ref. [11]). The results are shown in figs. 4–6. In the case of ^{9}Be, (fig. 4), the contributions to the t pick-up (for two values of the j-transfer), of the ^{5}He pick-up and their sum are represented. For ^{11}B, as discussed in sect. 2, two sets of optical-model parameters for the exit channel were used. The results with sets BA1 and BA2 are shown in figs. 5 and 6, respectively. Again the contributions of the separate mechanisms are plotted in addi-

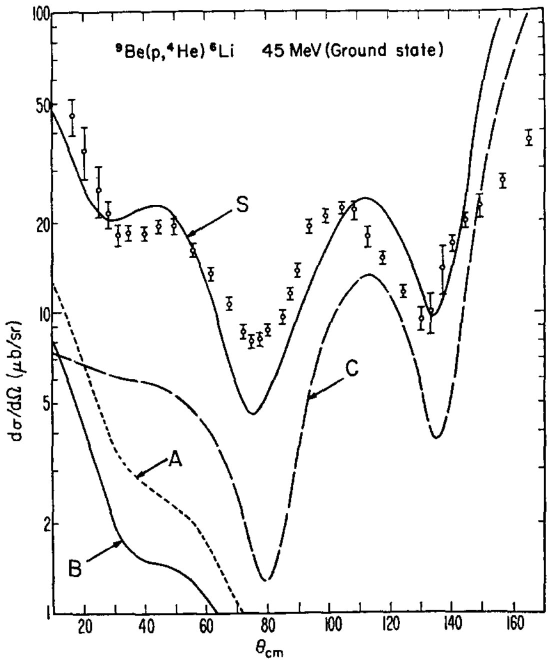

Fig. 4. Zero-range DWBA calculations for the reaction $^9Be(p, \alpha)^6Li$. The data are from ref. [1]); (A) t pick-up ($j = \frac{3}{2}$); (B) t pick-up ($j = \frac{1}{2}$); (C) ^5He pick-up ($s = \frac{3}{2}$; the magnitude of this curve should be multiplied by two); (S) total zero-range prediction.

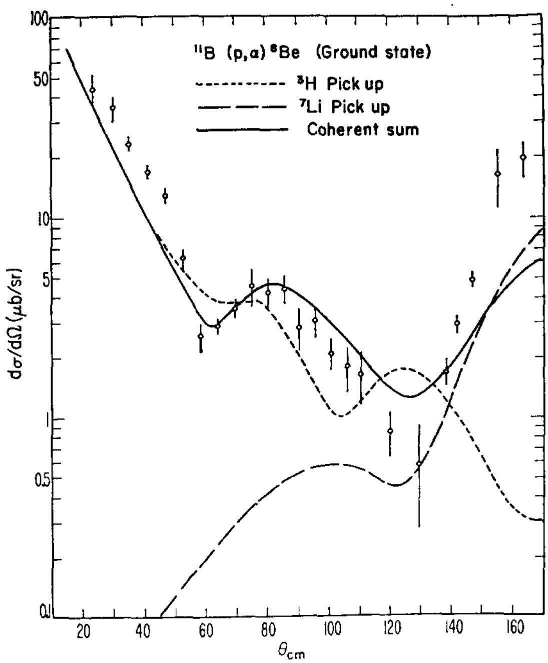

Fig. 5. Zero-range DWBA calculations for the $^{11}B(p, \alpha)^8Be$ reaction. The data are from ref. [1]). The optical-model set BA1 of table 3 was used.

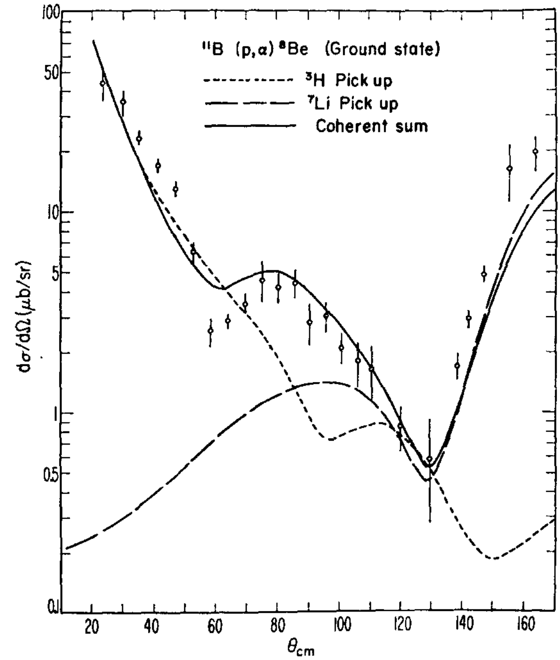

Fig. 6. See caption to fig. 5. The optical-model set BA2 of table 3 was used.

TABLE 4

Zero-range form-factor parameters; $R = r_0\ (A^{\frac{1}{3}}_{\text{core}}+A^{\frac{1}{3}}_{\text{particle}})$

	Bound-state quantum numbers	r_0	a_0
^{9}Be: (t+^{6}Li)	2p	0.95	0.80
^{9}Be: (^{5}He+^{4}He)	3s	1.59	0.20
^{11}B: (t+^{8}Be)	2p	0.87	0.30
^{11}B: (^{7}Li+^{4}He)	3s	1.09	1.00

tion to their sum. Concerning the summing of contributions, the following rules [11]) were applied: the addition is coherently done over different mechanisms and different values of the l- and s-transfer. In cases where different j-transfer values are allowed, the cross sections are added incoherently. The form-factor parameters are listed in table 4. The bound-state quantum numbers were determined on the basis of a simplified cluster picture of the nuclei [12]). First-order corrections are included to the zero-range DWBA matrix element in the case of the t pick-up. For both reactions, the finite-range parameter β was chosen as 1.36 fm^{-1}, as suggested in ref. [13]). This correction proved essential for obtaining the good fit to the ^{11}B data. In the case of ^{9}Be, the contribution of the t pick-up mechanism to the angular distribution is rather small (one of the most questionable results of the present zero-range analysis) and the introduction of the first-order correction hardly influences the fit.

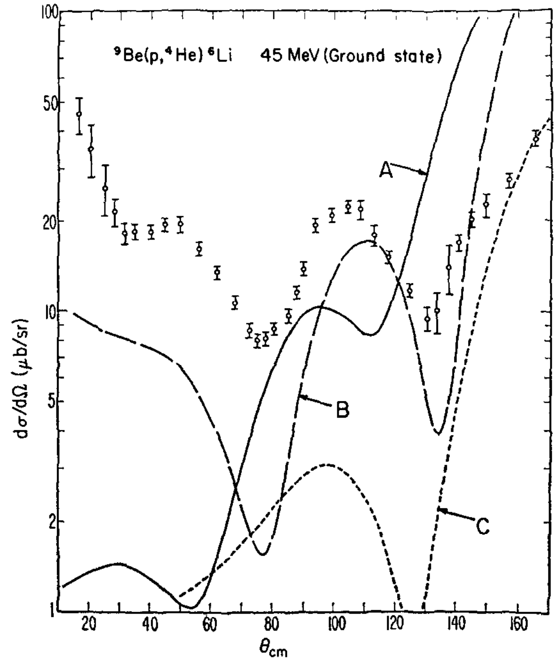

Fig. 7. Heavy-particle ^{5}He pick-up DWBA calculations for the reaction ^{9}Be(p, α)^{6}Li; (A) exact finite-range; (B) zero-range; (C) fixed-range approximation. The curves are arbitrarily scaled.

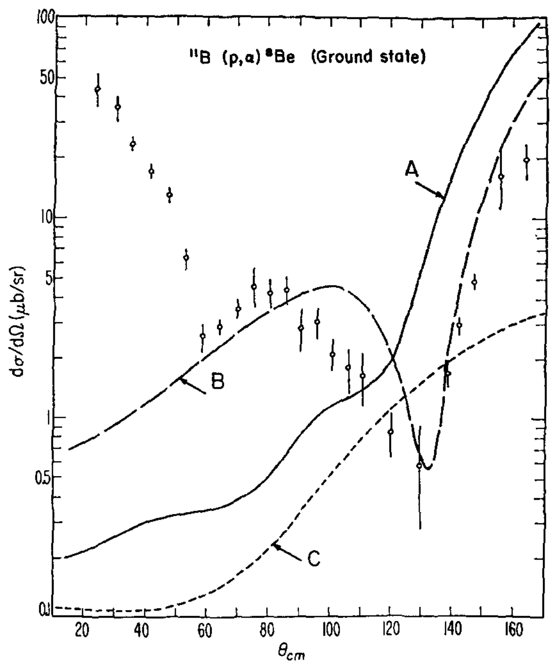

Fig. 8. Heavy-particle ^{7}Li pick-up DWBA calculations for the reaction ^{11}B(p, α)^{8}Be: (A) exact finite-range; (B) zero-range; (C) fixed-range approximation. The curves are arbitrarily scaled. Optical-model set BA2.

In conclusion, although the data can be rather well fit by zero-range estimations of the DWBA matrix element, the objections that one can formulate against such a treatment preclude one from attaching too much trust to the values of the parameters thus obtained. The exact finite-range kernels (see subsect. 3.3) indicate that, in the case of the heavy-particle pick-up particularly, the zero-range approximation selects a rather arbitrary and not necessarily representative region of the integration volume. The fact that for the ^{9}Be(p, α)^{6}Li reaction, the ^{5}He pick-up mechanism appears to dominate over the t pick-up in most of the angular range, is another indication that these results are rather meaningless and that even relative spectroscopic factors should not be trusted.

Zero-range estimates of the contribution from knock-on processes have been made as well [9]). In all cases, these contributions were found to be smaller by orders of magnitude if compared to the pick-up cross sections.

3.2. FIXED-RANGE DWBA CALCULATIONS

The zero-range approximation is particularly questionable in cases where the bound-state wave function on which the δ-function is applied is not narrowly confined to the origin. In both cases of the heavy-particle pick-up mechanisms considered here, the proton and the transferred cluster lie in a relative 1p state, of which both the radial and azimuthal aspects are ignored by the application of the zero-range condition. Puehlhofer *et al.* have proposed a "fixed-range" approximation [12]) to improve this situation. It implies in our case that the c.m. of the transferred cluster is kept at a certain fixed distance, the "range", from the incoming proton, but also that it is restricted to lie on the line connecting the proton and the residual nucleus. The results of such calculations with range parameters of the order of 0.5 fm are shown in figs. 7 and 8. The shapes of the angular distributions are not considerably changed by the introduction of this feature. The normalization of all curves shown in figs. 7 and 8 is arbitrary.

3.3. EXACT FINITE-RANGE ANALYSIS

The finite-range DWBA calculations have been performed by means of a computer code described previously [9,15]). The figs. 9 (for ^{9}Be) and 10 (for ^{11}B) show contour plots of the two-dimensional kernels $G_K(r_i, r_f)$ $(K = 0)$ for all pick-up mechanisms. The horizontal and vertical axes are scaled in all cases in such a way that the 45° diagonal represents the zero-range locus. The level scale is logarithmic, but negative numbers indicate negative, not small, kernels. Sign changes are related to nodes in the bound-state wave functions. These kernels, when multiplied by the proper angular momentum coefficients [16]), assume the role on the form factor in the double radial integration that has to be performed. The kernels for other values of K have similar spatial dependence, although their magnitudes decrease with increasing K. The bound-state parameters are listed in table 5. All finite-range calculations were performed without inclusion of spin-orbit forces.

TABLE 5

Finite-range form-factor parameters; $R = r_0\ (A^{\frac{1}{3}}_{\text{core}} + A^{\frac{1}{3}}_{\text{particle}})$

		r_0	a_0
^{9}Be:	(t+^{6}Li)	0.80	0.80
	(t+p)	0.90	0.50
^{9}Be:	(^{5}He+^{4}He)	1.80	0.20
	(^{5}He+p)	0.80	0.50
^{11}B:	(t+^{8}Be)	1.10	0.30
	(t+p)	0.90	0.50
^{11}B:	(^{7}Li+^{4}He)	1.30	0.20
	(^{7}Li+p)	0.99	0.50

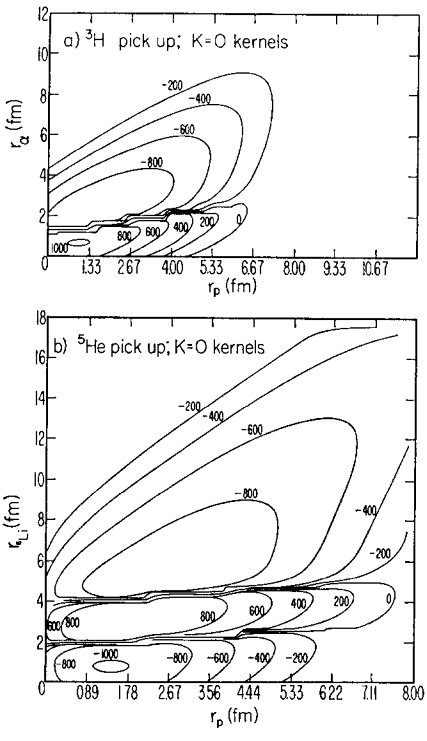

Fig. 9. Contour representation of the finite-range kernels $G_K(r_1, r_t)$ ($K = 0$) for the reaction ^{9}Be(p, α)^{6}Li. The scales on abscissae and ordinates are such that in all cases a 45° diagonal from the lower left corner represents the zero-range locus. The level scale is logarithmic and normalized at 1000 to the biggest kernel, but negative levels indicate negative, not small, kernels; (a) t pick-up kernels; (b) kernels for the ^{5}He pick-up.

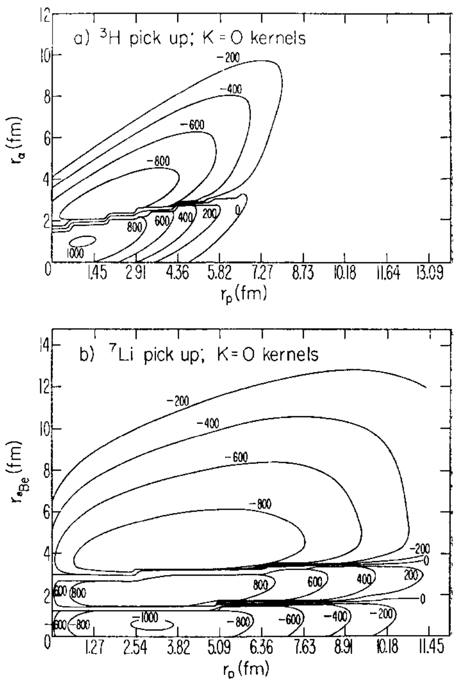

Fig. 10. Finite-range kernels for the ^{11}B(p, α)^{8}Be reaction. See caption to fig. 9; (a) t pick-up kernels; (b) kernels for the ^{7}Li pick-up.

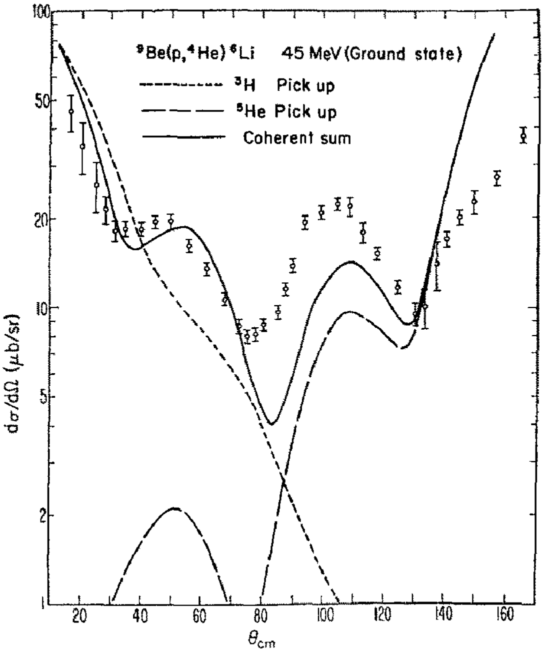

Fig. 11. Finite-range DWBA calculations for the reaction $^{9}Be(p, \alpha)^{6}Li$.

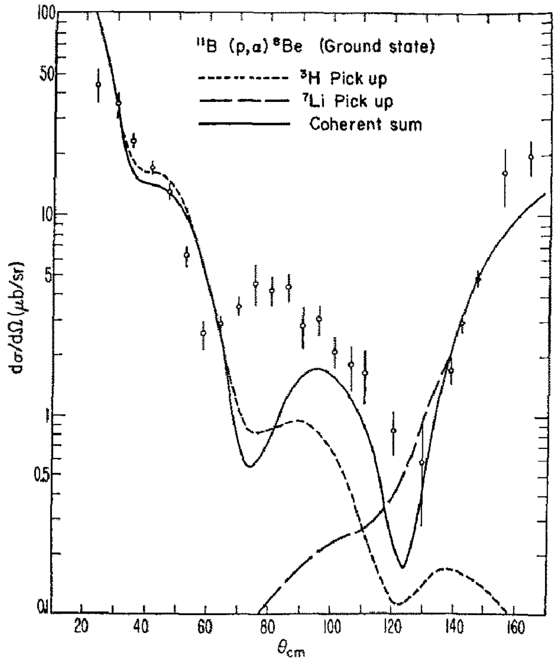

Fig. 12. Finite-range DWBA calculations for the reaction $^{11}B(p, \alpha)^{8}Be$. Optical-model set BA2.

The calculated cross sections are shown for both reactions in figs. 11 and 12, respectively, again separately for each mechanism and summed. Two features of those results ought to be underlined in a comparison with the zero-range curves. First the shapes are considerably less sensitive to a variation of the bound-state parameters. Secondly, in the case of ^{9}Be(p, α)^{6}Li, the relative importance of the two mechanisms appears more reasonable than before, a consequence of the fact that the ^{5}He pick-up now contributes considerably less at forward angles.

TABLE 6

Spectroscopic factors for cluster structures in the target nuclei

^{9}Be	$S(^9\text{Be} \rightleftarrows {}^6\text{Li}+\text{t}) = 0.003$ $S(^9\text{Be} \rightleftarrows {}^5\text{He}+\alpha) = 0.102$ ratio: 0.03
^{11}B	$S(^{11}\text{B} \rightleftarrows {}^8\text{Be}+\text{t}) = 0.768$ $S(^{11}\text{B} \rightleftarrows {}^7\text{Li}+\alpha) = 0.016$ ratio: 47.8

While the fits shown in figs. 11 and 12 are certainly not very good, they are, for the reasons mentioned above, more meaningful than the zero-range results. Presumably better sets of optical-model and bound-state parameters could be found. It should also be noted that no such free parameter as the finite-range correction is available to improve the fits and that no radial cut-off was used. It is, however, felt that an imperfect fit truly could indicate the limitations of the model, particularly of the cluster assumption mentioned in sect. 1.

All calculations presented here were performed in the post DWBA approximation. The choice, however, of the interaction potential in the matrix element presents a controversial feature. Usually only one, V_{bx} of the three terms of the exact expression [17]) $V_{bx}+V_{bA}-V_{bB}$ is included on the ground that the others approximately cancel each other. Smith [17]) has argued that only the real parts of those potentials approximately cancel, since one of the two terms is not an optical potential and is therefore real. An imaginary part of the interaction has consequently been included into the present calculation, with almost negligible change to the cross sections, however. It remains to be studied whether the effects of these two terms are adequately described by this approximation.

Finally, products of spectroscopic factors can be extracted by comparing theory and data. Assuming theoretical values for the final-state parentage [18]), one obtains spectroscopic factors for the cluster structures (α+^{5}He) and (t+^{6}Li) in ^{9}Be and (t+^{8}Be) and (α+^{7}Li) in ^{11}B, presented in table 6. In the present state of the analysis one should not consider these numbers to be more than estimates of the order of magnitude, which however appears quite reasonable.

4. Conclusion

Although the DWBA is frequently applied to the analysis of many-nucleon transfer reactions, its use presents many questionable aspects. Beyond doubt, cross sections should not be evaluated in the zero-range approximation. The choice of optical-model parameters, already difficult in the case of simple stripping reactions, is even more ambiguous here, particularly for the heavy-ion channel, where the situation is further complicated by the scarcity of available elastic scattering data. Finally, all such analyses assume that the reaction can be described by a simple three-body model, in which a tightly bound cluster is exchanged between two inert cores.

The fair fits that are obtained constitute the only justification, if at all, in favor of this last restriction in its extreme form, which however constitutes the only basis presently available on which calculations can be performed. Only a systematic survey of all the available data, including an analysis of heavy-ion scattering in terms of optical or other parametrizations, will indicate whether such a model yields a consistent and useful picture. If such is the case, it will provide a valuable test of predictions concerning the probability of presence of well-defined clusters in light nuclei.

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