Clusters in Nuclear Matter and the Equation of State

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Clusters in Nuclear Matter and the Equation of State

Stefan Typel

GSI Helmholtzzentrum für Schwerionenforschung, Planckstraße 1, D-64291 Darmstadt, Germany

E-mail: s.typel@gsi.de

Abstract. The equation of state of dense matter is an essential ingredient in astrophysical simulations, e.g. of core-collapse supernovae and neutron stars. At densities below nuclear saturation density, nucleon-nucleon correlations strongly affect the thermodynamical properties of matter. Clusters appear and the chemical composition is modified. These effects can be considered in a generalized relativistic density functional approach. It shows the correct limits by interpolating between the virial equation of state at low densities and quasiparticle mean-field models at high densities. Heavy-ion collisions offer the possibility to study the effects of correlations and the modification of cluster properties in dilute matter.

1. Introduction

During most of their lifetime, stars evolve steadily by burning light nuclei in fusion reactions creating heavier nuclei in the stellar core and surrounding shells. Massive stars ($M_{\mathrm{star}} \gtrsim 8 M_{\mathrm{sun}}$) develop an iron core that eventually collapses under its own gravitational attraction and a violent supernova explosion is launched [1]. The outer parts of the former star are ejected and a neutron star or black hole is left as a remnant. The dynamical evolution of such a core-collapse supernova and the properties of the neutron star are determined by the the equation of state (EoS) of dense stellar material [2]. It determines the thermodynamic conditions of the expanding matter that is the site of nucleosynthesis reactions. The chemical composition of the matter has an impact on the response to the copious amount of neutrinos emitted by the core. Since the timescale of the nuclear reactions is much shorter than that of the supernova evolution, an equation of state of dense matter in thermodynamical and chemical equilibrium can be used in astrophysical simulations assuming local charge neutrality.

Three parameters are sufficient to characterize the thermodynamic conditions of the system: the mass density ϱ (or baryon number density n), the temperature T and the electron fraction Y_e (or neutron-proton asymmetry $\beta = 1 - 2Y_e$). Typical ranges of their values in supernova simulations are $10^{-9} \lesssim \varrho/\varrho_{\mathrm{sat}} \lesssim 10$ (with the nuclear saturations density $\varrho_{\mathrm{sat}} \approx 2.5 \cdot 10^{14} \mathrm{g/cm^3}$), 0.1 MeV $\lesssim k_B T \lesssim$ 50 MeV and $0 \leq Y_e \lesssim 0.6$. Below ϱ_{sat} it is usually sufficient to consider nucleons, nuclei, electrons and thermal photons as the relevant degrees of freedom in stellar matter. Neutrinos are not included. They are treated independently of the EoS in simulations because they are not in equilibrium with the matter.

There are many EoS available in the literature, comprising a large variety of approaches from simple parametrizations to very elaborate models. Most investigations focus on particular aspects and properties of the matter for particular conditions, e.g. symmetric nuclear matter, neutron matter, matter in β equilibrium, low- or high-density matter et cetera. Since a global

description is required covering the full parameter space, only a small number of realistic EoS, usually in tabular form, is used in astrophysical simulations of core-collapse supernovae, see references [3, 4, 5] for the most well-known and widely applied EoS in astrophysical simulations and references [6, 7, 8, 9, 10, 11] for some recently developed or extended models. For the description of neutron stars, a zero-temperature EoS is usually appropriate and a much larger number of models can be employed.

The short-range nucleon-nucleon interaction modifies the thermodynamic properties of dense matter substantially. In particular, it is responsible for two-, three-, ..., many-body correlations that change the chemical composition at low densities by creating new particle species as bound states of nucleons. These clusters have to be considered explicitly in the description. Standard EoS tables consider only those particles that are also incorporated in astrophysical simulations, i.e. neutrons, protons, electrons, photons, the α particle and a representative heavy nucleus. In recent EoS, the set of particles was extended to include more light clusters (^{2}H, ^{3}H, ^{3}He, ^{4}He), a full distribution of all nuclei in the nuclear chart or exotic particles such as hyperons, other heavy baryons, mesons (pions, kaons, ...) or quarks.

At high densites around and above nuclear saturation density, nuclear matter is expected to be uniform and mean-field models with neutrons and protons as sole constituents are usually sufficient. Here, effects of the interaction are treated in a quasiparticle approach. At low densities and temperatures below a critical value of approximately 15 MeV nuclear matter will develop inhomogeneities with a coexistence of gas and liquid phases on a macroscopic scale. It has to be emphasized that this "liquid-gas" first-order phase transition is a feature of the fictitious system of nuclear matter where the Coulomb interaction is neglected and there are no electrons to enforce charge neutrality. In realistic stellar matter with Coulomb interaction and electrons, however, there is a formation of low- and high-density regions separated by various shapes of the interfaces ("pasta phases") on a microscopic scale. The transition is driven by the balance of the short-range nuclear and long-range Coulomb interaction. At very low temperatures, a solid phase with crystal/lattice structures develops. It is obvious that an appropriate construction of the phase transitions is needed and a consistent interpolations between the various regions is required in a global EoS for astrophysical applications. It is a great challenge to cover all aspects in a single model and often a combination of different approaches is required.

This contribution focuses on the theoretical description of dense matter at nuclear saturation density and (much) below, in particular the formation of clusters due to correlations and their dissolution with increasing density. In section 2 different concepts and approaches are discussed that allow to describe dense matter with clusters at low densities and that take more and more effects of the interaction into account. A generalized relativistic density functional (gRDF) approach is presented in section 3. It incorporates the effects of correlations and covers the whole density region up to nuclear saturation density with the correct low- and high-density limits. Medium effects on the properties of light and heavy nuclei within the gRDF approach are discussed in section 4. The low-density limit of the model is considered in section 5 and the effects on neutron matter are studied as an example. The behavior of light cluster abundances and the effects of two-nucleon scattering correlations are presented in section 6. The effects of cluster formation on the symmetry energy are investigated in section 7 and experimental tests of the model predictions are mentioned. Details and further references on the material presented in this contribution can be found in references [12, 13, 14]. In the following, natural units are used such that $\hbar = c = k_B = 1$ to simplify the equations.

2. Theoretical approaches

Density, temperature and neutron-proton asymmetry have a strong effect on the chemical composition and thermodynamic properties of dilute matter. There are essentially two different points of view that can serve as starting points to construct theoretical models of the system.

In the *chemical picture* one considers a mixture of nucleons and nuclei in chemical equilibrium. It is assumed that the properties of the particles composed of nucleons are independent of the medium, e.g. the binding energies of nuclei do not change with density or temperature. In order to take into account correlation effects, the interaction has to be specified between all different constituents individually. Describing the dissolution of the nuclei at high densities is a major problem. Usually, a simple geometric concept such as the excluded-volume mechanism is introduced.

In the *physical picture* only nucleons are considered as fundamental constituents. Their mutual interaction introduces correlations and leads to the formation of bound states and scattering resonances. Appropriate theoretical methods have to be developed to treat the two-, three-, ... many-body correlations inside the medium. The nucleon-nucleon interaction is the only and essential ingredient in this type of approach.

In the following, a number of theoretical approaches is presented that improve the description of nuclear matter considering different effects of the nuclear interaction. These models combine both pictures to various degrees.

2.1. Low-density models

2.1.1. Nuclear statistical equilibrium model The most simple approach to describe dilute nuclear matter is a nuclear statistical equilibrium (NSE) model that assumes an ideal mixture of nucleons (p, n) and nuclei (X) with mass numbers $A = N + Z$ in chemical equilibrium

$$Zp + Nn \Leftrightarrow {}^{A}_{Z}X \; . \tag{1}$$

Thus the nonrelativistic chemical potentials μ_i of the particles are related by

$$Z\mu_p + N\mu_n = \mu_X - B_X \tag{2}$$

with the binding energies $B_X > 0$ of the nuclei X with mass $m_X = Zm_p + Nm_n - B_X$. Mutual interactions are neglected. Assuming nonrelativistic kinematics and Maxwell-Boltzmann statistics, the grand canonical potential is given by the simple expression

$$\Omega(T, V, \mu_i) = -TV \sum_i \frac{g_i}{\lambda_i^3} \exp\left(\frac{\mu_i}{T}\right) \tag{3}$$

with thermal wavelenghts $\lambda_i = \sqrt{2\pi/(m_i T)}$ and degeneracy factors g_i. The summation over the index i comprises protons, neutrons and all nuclei. Excited states x of nuclei are usually taken into account by introducing temperature dependent degeneracy factors

$$g_i(T) = (2J_i^{gs} + 1) + \sum_x (2J_i^x + 1) \exp\left(-\frac{E_x}{T}\right) \tag{4}$$

with ground state and excited state spins J_i^{gs} and J_i^x, respectively. From Eq. (3) all thermodynamical properties, e.g. the equations of state, such as $pV = NT$ and $E = 3NT/2$ with $N = V \sum_i n_i$, for a mixture of ideal gases with partial densities n_i are derived. In particular, the individual particle number densities are given by

$$n_i = -\left.\frac{\partial \Omega}{\partial \mu_i}\right|_{T,V,\mu_{j\neq i}} = \frac{g_i}{\lambda_i^3} \exp\left(\frac{\mu_i}{T}\right) \; . \tag{5}$$

Considering the ratio $n_X/(n_p^Z n_n^N)$ for the reaction (1), one finds the law of mass action that is well known in chemistry.

The basic NSE model neglects all interactions between the constituents and cannot describe the dissolution of nuclei with increasing density. In order to include such an effect, the excluded-volume mechanism can be employed that suppresses the abundance of nuclei at high densities, see, e.g., reference [6].

2.1.2. Virial equation of state Two-, three-, ..., many-body correlations due to the interaction between the constituents can be incorporated in the description by using the virial equation of state (VEoS). In this approach the grand canonical partition function is expanded in powers of the particle fugacities $z_i = \exp(\mu_i/T)$. The expansion is only valid for small expansion parameters $z_i \approx n_i\lambda_i^3 \ll 1$ and thus can be applied only for low densities and not too low temperatures. Introducing the dimendionless cluster (virial) coefficients $b_i = g_i$, b_{ij}, ... the grand canonial potential in the virial expansion

$$\Omega(\mu_i, T, V) = -TV\left(\sum_i b_i \frac{z_i}{\lambda_i^3} + \sum_{ij} b_{ij} \frac{z_i z_j}{\lambda_i^{3/2}\lambda_j^{3/2}} + \dots\right) \tag{6}$$

is obtained. For independent particles without interaction, the second, third, ... virial coefficient vanish and the NSE result is recovered.

The effect of correlations between particles i and j is encoded in the second virial coefficient. In classical mechanics it is given by

$$b_{ij} = \frac{1}{2}\frac{g_{ij}}{\lambda_i^{3/2}\lambda_j^{3/2}} \int d^3 r_{ij} \left\{\exp\left[-\frac{V_{ij}(r_{ij})}{T}\right] - 1\right\} \tag{7}$$

with the two-body interaction potential V_{ij} depending on the distance r_{ij} and the degeneracy factor g_{ij} of the two-body state. G.E. Beth and E. Uhlenbeck derived the quantum mechanical generalization of b_{ij} [15, 16]. The integration over phase space in classical mechanics has to be replaced by a summation over quantum states. Then, the second virial assumes the form

$$b_{ij}(T) = \frac{1+\delta_{ij}}{2}\left(\frac{m_i + m_j}{\sqrt{m_i m_j}}\right)^{3/2} \int dE\, D_{ij}(E) \exp\left(-\frac{E}{T}\right) \pm \delta_{ij} g_i 2^{-5/2}\,. \tag{8}$$

The quantity

$$D_{ij}(E) = \sum_k g_k^{(ij)} \delta\left(E - E_k^{(ij)}\right) + \sum_l \frac{g_l^{(ij)}}{\pi}\frac{d\delta_l^{(ij)}}{dE} \tag{9}$$

can be seen as the difference of the level densities between the correlated and uncorrelated two-body system. It contains contributions from bound states at energies $E_k^{(ij)} < 0$ and scattering states with phase shifts $\delta_l^{(ij)}$ in channels k and l, respectively. The last term in (8) is a quantum statistical correction with positive (negative) sign if i and j are identical bosons (fermions).

In the two-nucleon system, the binding energy of the deuteron and the scattering phase shifts are known experimentally. Hence, b_{ij} depends only on measured data and the low-density behavior of the EoS can be established model-independently [17]. It can serve as a benchmark for other approaches. With increasing density, however, the power series approximation will fail. The dissolution of clusters such as the deuteron cannot be accounted for in the VEoS.

2.1.3. Quantum statistical model Medium effects on the properties of nuclei in matter can be included in a quantum statistical (QS) or generalized Beth-Uhlenbeck approach that was formulated using thermodynamic Green's functions [18]. The EoS of an interacting many-body system is derived from the total nucleon number density

$$n(\mu, T, V) = \sum_j \int \frac{dE}{2\pi} A(j, E) f_+(E) \tag{10}$$

with the spectral functions $A(j, E)$ for a single-particle state $j \equiv (\vec{p}_j, \sigma_j, \tau_j)$ and the thermal distribution function

$$f_\pm(E) = \{\exp[(E-\mu)/T] \pm 1\}^{-1} \tag{11}$$

for fermions (+) or bosons (−), respectively. The spectral function depends on the complex valued self-energy $\Sigma(j, z)$ that contains the information on the interaction. After expanding $A(j, E)$ for small imaginary parts of the self-energy, the total density splits into two parts $n = n_{\rm free} + 2n_{\rm corr}$. The first contribution $n_{\rm free} = \sum_j f_+[e(j)]$ is the density of free quasiparticles with quasiparticle energies $e(j)$. They are solutions of $e(j) = p_j^2/(2m_j) + {\rm Re}\Sigma[j, e(j)]$ and depend on the momentum in general. The correlation density

$$n_{\rm corr} = \sum_k g_k^{(2)} \sum_{\vec{P}, P>P_{\rm Mott}} f_-(E_{\rm cont} - B_k) + \sum_l g_l^{(2)} \sum_{\vec{P}} \int \frac{dE}{\pi} 2\sin^2\delta_l \frac{d\delta_l}{dE} f_-(E_{\rm cont} + E) \tag{12}$$

receives contributions from bound states and from scattering states. This form resembles the structure of the second virial coefficient (8) in the VEoS. In contrast to b_{ij}, the properties of the two-body states in the QS model depend on their c.m. momentum $\vec{P}$ with respect to the medium. Due to the action of the Pauli principle that blocks states by the medium, there are no two-body bound states for P below the Mott momentum $P_{\rm Mott}$. The continuum edge $E_{\rm cont}$ is defined as the energy of the scattering states with zero relative momentum. The binding energies B_k and the in-medium scattering phase shifts δ_l depend on the medium properties and $\vec{P}$. They have to be determined from the in-medium T matrix in general. The Bose-Einstein distribution functions f_- appears correctly for the two-nucleon states. There is an additional $2\sin^2\delta_l$ factor in the continuum contribution as compared to the integral in equation (8). It reduces the strength of the explicit scattering correlations since the self-energies contain a part of the correlation effect due to the interaction.

There are different ways to determine the medium-dependent shift of the nuclear binding energies. For light nuclei with mass number $A \leq 4$ they can be calculated by solving the appropriate in-medium Schrödinger equation for the composite system with realistic nucleon-nucleon potentials or with nucleon self-energies taken from phenomenological mean-field models. Results are discussed in subsection 4.1. For heavier nuclei ($A > 4$) a different approach based on a Wigner-Seitz cell calculation with an energy density functional can be used, see subsection 4.2.

2.2. Intermediate/high-density models

Two major classes of theoretical approaches can be distinguished that describe nuclear matter successfully around the nuclear saturation density. There are phenomenological approaches with effective interactions, e.g. nonrelativistic Hartree-Fock models with the Skyrme or Gogny interaction and relativistic mean-field models, and there are "ab-initio" approaches that use realistic nucleon-nucleon interactions fitted to two- (and three-) nucleon bound and scattering states, e.g. (non-)relativistic Brueckner-Hartree-Fock calculations. Results of the phenomenological models depend on a small number of parameters that are usually determined by fits to properties of finite nuclei. In this way, shortcomings of the model and the approximations in the many-body approach can be counterbalanced. In constrast, the interaction in the "ab-initio" models is given from the outset and the quality of the results is determined by the sophistication of the many-body method. Unfortunately, the latter models have some limitations in their application, e.g. typically a restriction to uniform nuclear matter.

In order to provide an EoS for astrophysical applications that is applicable for a wide range of temperature, density and neutron-proton asymmetry, mean-field models are the prevailing choice in order to obtain quantitatively reasonable results. These models can be formulated in

the language of energy density functionals with well-known techniques for deriving and solving the relevant equations.

Relativistic approaches are preferred by several reasons. Lorentz covariance is a fundamental principle in physics constraining the form of the possible interactions. The spin of nucleon is naturally included in the relativistic description and the spin-orbit interaction appears with the correct strength in nuclei. At high densities, relativistic effects are important due to the high Fermi momenta of the nucleons and a superluminal speed of sound will not occur in these models. Beyond the kinematical effects, strong vector and scalar potentials appear in nuclear matter that are responsible for a new saturation mechanism. The mathematical formulation is quite elegant with a close connection to models based on quark degrees of freedom.

Despite many virtues, mean-field models with only nucleons as basic constituents fail at low densities. Explicit correlations are not considered and the formation and dissolution of nuclei is not incorporated into the description.

3. Generalized relativistic density functional

In conventional relativistic mean-field (RMF) models, neutrons and protons are the fundamental degrees of freedom. They are treated as quasiparticles with self-energies that depend on the medium properties and take into account the effects of the interaction. Electrons (and myons) can be added in astrophysical applications when the EoS of charge neutral stellar matter is needed and the Coulomb interaction has to be included. The nuclear interaction is modeled by an exchange of mesons (usually ω, σ, ρ, δ) that couple minimally to the nucleons. They are represented by boson fields that are generally treated as classical fields. Similarly, photons can be represented by the corresponding electromagnetic potential.

The idea of the generalized relativistic density functional (gRDF) approach is to introduce new degrees of freedom in the description in order to include nuclei as constituents and to consider correlations in an effective way. In reference [12] the Lagrangian density of a RMF model was extended by introducing light nuclei (^{2}H, ^{3}H, ^{3}He and ^{4}He). They are represented by corresponding spin 1, 1/2 and 0 fields. Nuclei interact with the meson and photon fields like the nucleons, only the couplings are rescaled with the appropriate factors related to their neutron and proton content. In addition, it is assumed that the binding energies of the nuclei are medium dependent quantities. In the particular approach of reference [12], this dependence enters through functions of the temperature and vector meson fields. Nucleon-nucleon scattering correlations, i.e. the nn, np and pp scattering states, were added to the model in reference [13]. The complete positive energy continuum in a particular channel is represented by a single state with an effective "resonance" energy. This energy depends on the medium properties like the binding energy of nuclei. The degeneracy factor of the effective continuum state depends on temperature. See section 5 for details. This corresponds to the treatment of excited states in nuclei in NSE models in subsection 2.1.1. The coupling to the meson and photon fields is modeled in the same way as for nuclei.

Finally, heavy nuclei with mass number $A > 4$ can be incorporated in a similar way as light nuclei. However, their binding energy shifts in the medium have to be determined using a different strategy, see subsection 4.2 for details and first results.

In practical applications of RMF models it is not sufficient for a quantitative description to assume minimal nucleon-meson couplings with constant strength Γ_{im} between particles i and mesons m. In order to incorporate a medium dependence of the effective nuclear interaction, a modification of the model is needed. One approach uses additional terms in the Lagrangian density with nonlinear self-interactions of the mesons. Usually, a polynomial dependence on various combinations of the meson fields is introduced. However, stability problems can occur for certain parametrizations and an extrapolation of the model to the high-density region has to be considered with care. In a second approach, motivated by results of Dirac-Brueckner

calculations of nuclear matter [19], the couplings Γ_{ik} are assumed to depend on the medium density. The functional form of the dependence can be chosen flexibly with a well controlled asymptotic behavior. The gRDF approach uses density dependent meson-nucleon couplings.

The number of free parameters in phenomenological RMF models for nucleonic matter is rather small. They are determined in fits to properties of nuclei and nuclear matter parameters that can be extracted from experiments. Values for the particle masses are usually close to experimental numbers with the exception of the σ meson that is not well constraint from measurements. The meson-nucleon couplings are specified at a certain reference density ϱ_{ref} and a simple functional form of their density dependence with few parameters is introduced. When nuclei are introduced in the gRDF approach, additional parameters need to be specified that determine the medium dependence of the binding energies. Besides constraints from nuclear physics, a comparison of theoretical predictions with astronomical observations can be used to test EoS models, see, e.g., reference [20].

The Lagrangian formulation of the RMF models can be converted to a density functional form that serves as a starting point to derive all relevant equations and thermodynamical quantities. In the present application, the grand canonical thermodynamical potential

$$\Omega = \int d^3r\, \omega_g(T, \mu_i, \sigma, \delta, \omega_0, \rho_0, \vec{\nabla}\sigma, \vec{\nabla}\delta, \vec{\nabla}\omega_0, \vec{\nabla}\rho_0, \vec{\nabla}A) \tag{13}$$

with a potential density ω_g is very convenient. The temperature T, the chemical potentials μ_i of all particles i, the meson and photon fields σ, δ, ω_0, ρ_0, A and their derivatives are the independent variables in this approach. The gRDF includes nucleons, bound states of nuclei and two-nucleon scattering states in the baryonic sector. Their masses are given by

$$m_i = N_i m_n + Z_i m_p - (1-\delta_{in})(1-\delta_{ip}) B_i^{(\mathrm{vac})} \,. \tag{14}$$

with the vacuum binding energy $B_i^{(\mathrm{vac})} \geq 0$ of the composite systems ($B_i^{(\mathrm{vac})} = 0$ for the scattering states). The energy of a baryon or lepton has the relativistic form

$$E_i = \sqrt{k^2 + (m_i - S_i)^2} + V_i \tag{15}$$

with scalar potentials

$$S_i = \Gamma_{i\sigma}\sigma + \Gamma_{i\delta}\delta - (1-\delta_{in})(1-\delta_{ip})\Delta B_i \tag{16}$$

and vector potentials

$$V_i = \Gamma_{i\omega}\omega_0 + \Gamma_{i\rho}\rho_0 + \Gamma_{i\gamma}A_0 + (\delta_{in}+\delta_{ip})V^r \tag{17}$$

that depend on the meson fields σ, δ, ω_0, ρ_0 and the photon field A. The scalar self-energy (16) contains the medium dependent binding energy shift $\Delta B_i \geq 0$. The rearrangment contribution

$$V^r = \Gamma'_\omega \omega_0 n_\omega + \Gamma'_\rho \rho_0 n_\rho - \Gamma'_\sigma \sigma n_\sigma - \Gamma'_\delta \delta n_\delta \tag{18}$$

appears in the vector self-energy. This term is due to the density dependence of the couplings $\Gamma_{ik} = g_{ik}\Gamma_k(\varrho)$ with scaling factors $g_{i\omega} = g_{i\sigma} = N_i + Z_i$, $g_{i\rho} = g_{i\delta} = N_i - Z_i$, $g_{i\gamma} = Z_i$ and $\varrho = n_n + n_p$. Neglecting antiparticle and boson condensate contributions, the potential density in equation (13) can be written as

$$\begin{aligned}
\omega_g \;=\; & \sum_i \mp g_i T \int \frac{d^3k}{(2\pi)^3} \ln\left[1 \pm \exp\left(-\frac{E_i - \tilde{\mu}_i}{T}\right)\right] \\
& -\frac{1}{2}\left(m_\omega^2\omega_0^2 + m_\rho^2\rho_0^2 - m_\sigma^2\sigma^2 - m_\delta^2\delta^2\right. \\
& \left. +\vec{\nabla}\omega_0\cdot\vec{\nabla}\omega_0 + \vec{\nabla}\rho_0\cdot\vec{\nabla}\rho_0 - \vec{\nabla}\sigma\cdot\vec{\nabla}\sigma - \vec{\nabla}\delta\cdot\vec{\nabla}\delta + \vec{\nabla}A_0\cdot\vec{\nabla}A_0\right) \\
& + \left(\Gamma'_\omega\omega_0 n_\omega + \Gamma'_\rho\rho_0 n_\rho - \Gamma'_\sigma\sigma n_\sigma - \Gamma'_\delta\delta n_\delta\right)(n_n + n_p)
\end{aligned} \tag{19}$$

where the upper (lower) signs applies to fermions (bosons). The source densities in equations (18) and (19) are determined by $n_k = \sum_i g_{ik} n_i$ for $k = \omega, \rho$ and $n_k = \sum_i g_{ik} n_i^s$ for $k = \sigma, \delta$, respectively, with the individual vector densities

$$n_i = g_i \int \frac{d^3k}{(2\pi)^3} \left[\exp\left(\frac{E_i - \tilde{\mu}_i}{T} \right) \pm 1 \right]^{-1} \tag{20}$$

and scalar densities

$$n_i^s = g_i \int \frac{d^3k}{(2\pi)^3} \frac{m_i - S_i}{\sqrt{k^2 + (m_i - S_i)^2}} \left[\exp\left(\frac{E_i - \tilde{\mu}_i}{T} \right) \pm 1 \right]^{-1} . \tag{21}$$

The relativistic chemical potentials of the baryonic particles are given by

$$\tilde{\mu}_i = N_i \tilde{\mu}_n + Z_i \tilde{\mu}_p = \mu_i + m_i \tag{22}$$

and the degeneracy factors g_i can depend on the temperature T. Note that n_i and n_i^s have to be considered as functions of the independent variables of Ω.

All field equations can be derived in the usual way from ω_g. They include rearrangement contributions that ensure the thermodynamical consistency of the model. E.g. the temperature dependence of the binding energy shifts and degeneracy factors generates contributions to the entropy, see reference [13] for details. The set of field equations has to be solved self-consistently in order to determine the two independent chemical potentials μ_n and μ_p for given total nucleon density n and asymmetry $\beta = 1 - 2Y_p$ with total proton fraction Y_p.

4. Medium effects on nuclear binding energies

The essential feature of the gRDF approach is the medium dependence of the nuclear binding energies causing the suppression of the cluster abundances with increasing density. The main origin of this shift is the action of the Pauli principle. Nucleons of the background medium occupy low-momentum states that are no longer available for the formation of clusters. An increase of the temperature reduces the blocking effect due to the increased diffuseness of the Fermi distribution.

4.1. Light nuclei

For light clusters ($A \leq 4$), the medium dependent shifts of the binding energies in the gRDF approach are adopted from the results of the QS model. They are parametrized as a function of temperature and an effective density that is reconstructed from the vector meson fields, see reference [12] for details. The density dependence of the binding energies for constant temperature are depicted in figure 1 of reference [12]. At zero density of the medium, the binding energy of a cluster is given by the experimental value in vacuum. With increasing density, the binding energy becomes smaller crossing the zero line at a certain point depending on T. This indicates that the cluster becomes unbound. Different extrapolations to high densities were explored in reference [13]. More recent parametrizations of the energy shifts, including the momentum dependence, can be found in reference [21].

4.2. Heavy nuclei

The binding energy shifts of heavy clusters are calculated on the basis of the relativistic density functional. Thus no additional model parameters are required. For this purpose, the formation of heavy clusters is modeled by spherical Wigner-Seitz cell calculations with an inhomogeneous density distribution of nucleons and electrons under the condition of a vanishing total charge. An

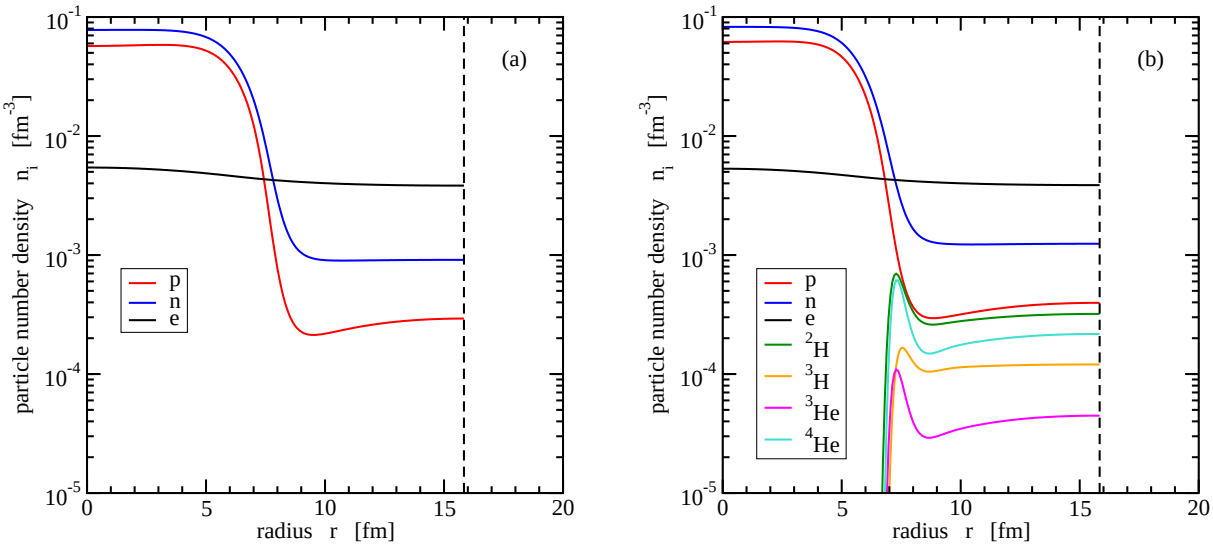

Figure 1. Density distribution of particles in a spherical Wigner-Seitz cell from a gRDF calculation in an extended Thomas-Fermi approximation without (a) and with (b) light clusters for temperature $T = 5$ MeV, total nucleon density $n = 0.01$ fm^{-3} and proton fraction $Y_p = 0.4$. Figure adapted from reference [14].

extended Thomas-Fermi approximation is used to to solve the field equations self-consistently with all particles interacting. No particular shape of the density distributions is assumed as, e.g. in the EoS calculations in references [3, 4]. The model goes beyond a simple local density approximation because the density functional takes into account the finite range of the interactions. However, shell effects are neglected.

For given temperature $T = 5$ MeV, total density $n = 0.01$ fm^{-3} and total proton fraction $Y_p = 0.4$, the density distribution of particles inside the Wigner-Seitz cell is depicted in figure 1. Nucleons form a heavy cluster in the center of the cell that is surrounded by a gas of nucleons. Electrons are almost uniformly distributed inside the cell since they form a highly degenerate Fermi gas. They effectively screen the Coulomb potential of the protons resulting in a vanishing potential on the surface of the cell consistent with the total charge neutrality.

When light clusters are considered in the calculation in addition to nucleons and electrons, an interesting effect is observed, see the right panel of figure 1. Due to the high density inside the heavy nucleus, light clusters can only appear in the in the surrounding low-density nucleon gas. There is an enhancement of the light cluster abundancies at the radius of the heavy cluster. This could be an indication of strong few-nucleon correlations on the nuclear surface.

The binding energy of a nucleus at a certain density of the medium can be determined by comparing the energies of the Wigner-Seiz cell calculation with uniform and nonuniform density distributions. In the limit of zero total density, the radius of the cell approaches infinity and the result with the inhomogeneous particle distributions corresponds to that of a nucleus surrounded by a cloud of electrons, a neutral atom. Since the extended Thomas-Fermi approximation does not take into account shell effects, it is more reasonable to extract only relative shifts of the binding energies from the Wigner-Seitz cell calculations than to use the absolute binding energies inside the medium. Correspondingly, the vacuum binding energies from experiment or mass tables have to be added to these shifts.

Experimental binding energies per nucleon [22] as a function of the mass number A are depicted in in figure 2 by black points. The maximum near $A = 60$ and shell effects close to doubly magic nuclei are clearly visible. When the binding energy shifts in the medium of a

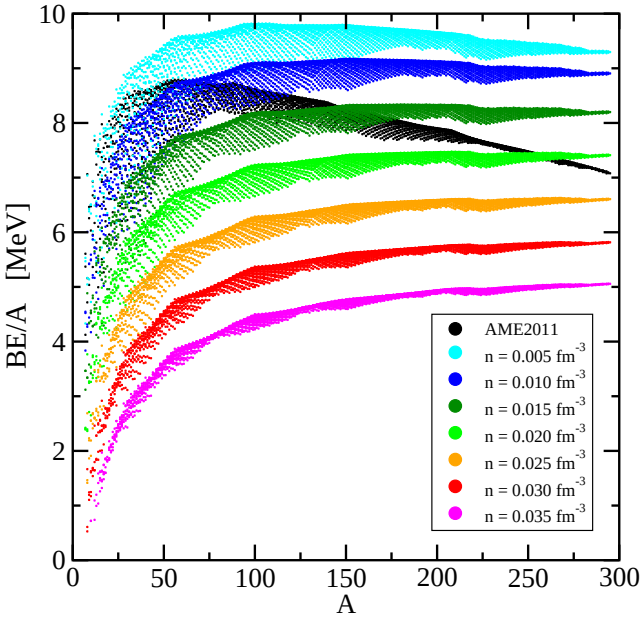

Figure 2. Binding energy per nucleon of nuclei with mass number A in vacuum (AME2011 [22]) and in the medium at temperature $T = 0$ MeV for various densities n. Figure from reference [14].

certain density are taken into account, a clear change in the A dependence is observed. At low medium densities, the screening of the Coulomb field by the electrons causes a stronger binding of the nuclei. The effect is stronger for nuclei with large charge numbers. In contrast, at higher medium densities, the binding energies of the nuclei reduce substantially because there is less increase of binding energy when a cluster is formed from a uniform nucleon distribution. Light clusters dissolve at lower medium densities than heavier clusters. The slope of the distribution and the position of the most strongly bound nuclei change with the medium density, too.

Traditional EoS tables for astrophysical applications, see, e.g. Refs. [3, 4], consider only one representative nucleus, the so-called single-nucleus approximation (SNA). Only more recent NSE-type models [6, 7] cover the the full distribution of nuclei. These models usually take into account a correction of the binding energy due to the Coulomb screening effect but the reduction of the nuclear binding energies at higher medium densities is generally not considered. A significant change in the abundance distribution can be expected from the gRDF calculations and it remains to be seen how this effect modifies the results of astrophysical simulations.

5. Low-density limit

At low densities and finite temperatures, the VEoS represents the correct description of the matter properties because only two-body correlations are relevant. Thus, the EoS in the gRDF approach should reproduce the model-independent low-density results of the VEoS. Consistency relations can be derived by comparing the fugacity expansions of the grand thermodynamical potential Ω in both models. They can be used to determine the effective resonance energies E_{ij} and degeneracy factors of the nucleon-nucleon scattering states $g_{ij}^{(\mathrm{eff})}$ that both depend on T. One finds that there are already corrections to the first order coefficient b_i in equation (13) of the VEoS due to the relativistic kinematics.

Introducing the virial integrals for continuum correlations of particles i and j

$$I_l^{(ij)} = \int \frac{dE}{\pi} \frac{d\delta_l^{(ij)}}{dE} \exp\left(-\frac{E}{T}\right) , \tag{23}$$

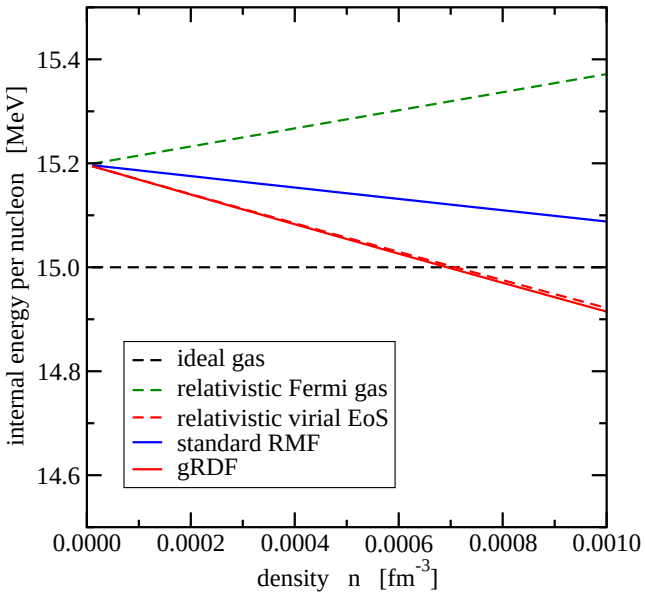

Figure 3. Internal energy per nucleon for neutron matter at temperature $T = 10$ MeV as a function of the total neutron density n in different approximations. Figure adapted from reference [14].

the contribution from different scattering channels can be collected in a single exponential term

$$\sum_l g_l^{(ij)} I_l^{(ij)} = \pm g_0^{(ij)} \exp\left[-\frac{E_{ij}(T)}{T}\right] \tag{24}$$

that defines $E_{ij}(T)$. The sign on the right-hand side of equation (24) is determined uniquely by the sign of the left-hand side. The effective resonance energies usually increase smoothly with temperature as depicted in figure 2 of reference [13]. At low temperatures, only the s-wave channel contributes substantially to the virial integral and the effective-range expansion of the phase shift can be used to obtain analytical results for the virial integrals.

Neglecting relativistic correction, the consistency relations

$$\frac{1}{\lambda_{nn}^3} \sum_l g_l^{(nn)} I_l^{(nn)} = \frac{1}{\lambda_{nn}^3} g_{nn}^{(\mathrm{eff})}(T) \exp\left[-\frac{E_{nn}(T)}{T}\right] - \frac{g_n^2}{\lambda_n^6} \frac{C_+}{2T} \tag{25}$$

for the nn channel and

$$\frac{1}{\lambda_{np}^3} \sum_l g_l^{(np)} I_l^{(np)} = \frac{1}{\lambda_{np}^3} \sum_{t=0}^{1} g_{npt}^{(\mathrm{eff})}(T) \exp\left[-\frac{E_{npt}(T)}{T}\right] - \frac{g_n g_p}{\lambda_n^3 \lambda_p^3} \frac{C_-}{T} \tag{26}$$

for the np channel are found from a comparison of the second virial coefficients in the VEoS and gRDF approaches. The sum of the two possible isospin channels $t = 0, 1$ is explicitly indicated in equation (26). The occurrence of a term that depends on the meson couplings Γ_k at zero density through the coefficients $C_\pm = C_\omega - C_\sigma \pm C_\rho \mp C_\delta$ with $C_k = [\Gamma_k(0)]^2/m_k^2$ is the important feature in these relations. Since the effective resonance energies $E_{ij}(T)$ are already defined by relation (24), equations (25) and (26) serve as the definition of the effective degeneracy factors $g_{nn}^{(\mathrm{eff})}(T)$ and $g_{npt}^{(\mathrm{eff})}(T)$. Again, a smooth dependence on the temperature is found as depicted in figure of reference [13].

In the zero-temperature limit, the consistency relations simplify substantially and two conditions are found that connect the differences $C_\omega - C_\sigma$ and $C_\rho - C_\delta$ with the scattering lengths of the relevant four nucleon-nucleon s-wave scattering channels. Conventional RMF parametrizations violate these relations, see references [13, 14].

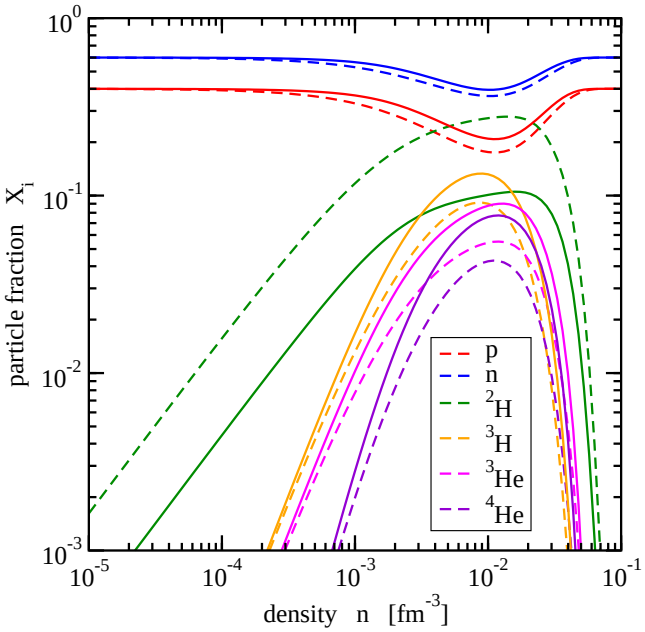

Figure 4. Particle fractions $X_i = n_i/n$ in nuclear matter with temperature $T = 10$ MeV and total proton fraction $Y_p = 0.4$ as a function of the total nucleon density n in the gRDF approach without heavy clusters. Calculation without (dashed lines) and with (full lines) nucleon-nucleon scattering correlations. Heavy clusters with mass number $A > 4$ are not considered in this calculation. Figure adapted from reference [14].

In pure neutron matter, various effects on the low-density EoS are most easily compared. There is no bound state and two-nucleon correlations appear only in the nn scattering channel. In figure 3 the internal energy per neutron E/N is depicted as a function of the total neutron density in different approximations for $T = 10$ MeV. For a nonrelativistic ideal gas, it is just $E/N = 3T/2 = 15$ MeV independent of n. The internal energy per nucleon at zero density ist shifted to higher values by relativistic effects. With quantum statistical corrections of a Fermi gas, an increase of E/N with the density is observed. The exact VEoS exhibits a different slope due to the considered correlations but a standard RMF calculation does not reproduce this behavior. In constrast, the gRDF model with the constribution of the effective nn scattering correlation perfectly reproduces the VEoS at low densities.

6. Light clusters and continuum correlations

At densities below nuclear saturation density, the composition of matter changes substantially when the thermodynamical variables T, n and Y_p are varied. The evolution of the particle mass number fractions $X_i = A_i n_i/n$ is shown in figure 4 as a function of the total nucleon density n for $T = 10$ MeV and $Y_p = 0.4$ for the gRDF model using the parametrization DD2 [12]. A calculation without nucleon-nucleon continuum correlations (dashed lines) is compared to a calculation with these correlations (full lines). Protons and neutrons dominate the composition of matter at very low densities with a tiny fraction of deuterons. The abundance of heavier clusters is negligible at these densities. With increasing density, cluster contributions to the composition become more important. First, three-body (^{3}H, ^{3}He) and then four-body (^{4}He) correlations appear and the number of free nucleons is reduced. The total cluster fraction reaches a maximum at approximately 1/10 of the nuclear saturation density. Increasing the density further causes a reduction of the cluster abundancies and finally they disappear. Thus the gRDF model can describe the dissolution of clusters in matter, i.e. the Mott effect, by assuming a medium dependence of the cluster properties, more precisely the binding energies.

In figure 4 a substantial effect of two-nucleon scattering correlations on the cluster fractions can be observed. They lead to a reduction of the number of deuteron-like correlations and a redistribution of the remaining particle fractions. The particle fractions in the gRDF model correspond to those those of quasiparticles and not to those of the original constituents. The

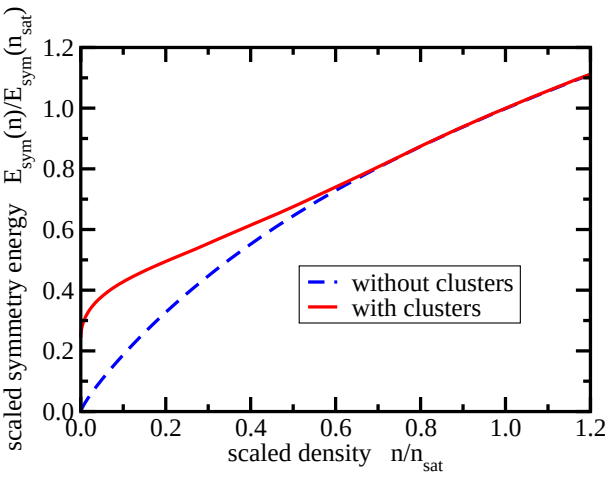

Figure 5. Scaled symmetry energy of nuclear matter at zero temperature as function of the scaled density in the gRDF model with parametrization DD2 without (blue dashed line) and with (red full line) cluster formation taken into account. Figure adapted from reference [14].

finite self-energy of the quasiparticles accounts for some part of the correlation strength due to the interaction and the size of explicit correlations is reduced.

7. Symmetry energy

The density dependence of the nuclear matter symmetry energy $E_{\rm sym}(n)$ is currently investigated with much experimental and theoretical effort, see references [23, 24] for constraints and the importance in nuclear physics and in astrophysical applications. Usually, $E_{\rm sym}(n)$ is defined as the second derivative of the energy per nucleon $E(n,\beta)/A$ in nuclear matter with respect to the neutron-proton asymmetry β, thus representing the curvature of the energy per nucleon in the direction of isospin asymmetry. In many cases, a quadratic approximation is sufficiently precise and the symmetry energy can be obtained by the finite-difference formula

$$E_{\rm sym}(n) = \frac{1}{2}\left[\frac{E}{A}(n,1) - 2\frac{E}{A}(n,0) + \frac{E}{A}(n,-1)\right] \tag{27}$$

that compares symmetric nuclear matter ($\beta = 0$) with pure neutron (proton) matter ($\beta = \pm 1$). At finite temperatures, one has to distingish the internal symmetry energy $E_{\rm sym}$ and the free symmetry energy $F_{\rm sym}$.

The density dependence of $E_{\rm sym}$ at low densities is strongly modified by the appearance of clusters, see figure 5 for matter at zero temperature. $E_{\rm sym}(n)$ approaches zero in the limit of vanishing density n in conventional mean-field models of uniform matter without cluster degrees of freedom. When the formation of clusters is taken into account, the system becomes more bound, in particularly for symmetric nuclear matter and at low temperatures. In contrast, the energy per nucleon of neutron matter is much less affected by correlations. As a consequence, the symmetry energies rises as compared to that in mean-field models without correlations. For vanishing temperature it even approaches a finite value in the limit of zero density. At higher temperatures, the effects will be less pronounced but possibly still detectable in experiments.

It has to be emphasized that the symmetry energy (27), extracted from the total energies per nucleon, contains also Coulomb contributions if clusters are taken into account. This is in contrast to the Bethe-Weizsäcker mass formula for nuclei where Coulomb and symmetry energy contributions are considered separately. In principle, the electromagnetic contribution to the total energy can be extracted unambiguously from the model calculations and the density dependence of the pure nuclear symmetry energy can be determined.

The analysis of heavy-ion collisions can help to observe the correlation effects on the symmetry energy by determining the thermodynamical properties of the expanding system as complete

as possible. There are strong experimental efforts to extract the symmetry energy and to study the properties of fragments in dilute matter. Recent results indicate an increase of the symmetry energy as predicted by models with correlations and cluster formation, see references [25, 26, 27, 28, 29] for details.

8. Conclusions

The properties of interacting many-body systems, in particular nuclear matter, are strongly affected by correlations. The chemical composition can change by forming clusters, i.e. many-body correlations in bound and scattering states, and their dissolution at high medium densities. This aspect has to be taken into account in theoretical models for the equation of state because thermodynamical properties of dense matter are modified by these changes. Correlation effects will also have consequences in applications such as astrophysical simulations of core-collapse supernovae or neutron star models. It is a great challenge to cover the relevant range in the thermodynamical variables temperature, density and isospin asymmetry in a single theoretical model.

Nuclear matter can be described by a number of theoretical approaches that consider correlations on various levels of sophistication using different concepts. At densities near nuclear saturation, properties of dense matter are quite successfully described by mean-field models where nucleons are considered as quasiparticles. Their self-energies depend on the medium and incorporate the effect of correlations, however explicit correlations are not taken into account. At low densities, several approaches to construct on EoS were devised in the past. Simple nuclear statistical equilibrium models are based on an ideal mixture of nucleons and nuclei without taking into account the interaction between these constituents. A suppression of clusters can be incorporated into the model using a geometrical excluded volume mechanism. Two-body correlations are explicitly included in the virial equation of state that serves as a model independent benchmark at low densities and not too low temperatures since the results depend only on experimental data. At high densities this approach fails because it is based on a series expansion in small particle fugacities and medium effects on the cluster properties are neglected. In a quantum statistical/generalized Beth-Uhlenbeck approach, these effects can be included on a microscopic level leading to the dissolution of clusters when their binding energies vanish.

A generalized relativistic density functional approach provides an interpolation between the correct low-density and high-density limits. This model considers nucleons, two-nucleon continuum states and nuclei as degrees of freedom. These constituents are treated as quasiparticles with medium-dependent self-energies. The nuclear interaction is described by an exchange of mesons with density dependent couplings to the nucleons, either free or bound in clusters. The model parameters are well determined by fitting to properties of finite nuclei and nuclear matter. Clusters, i.e. composite particles in bound or scattering states, change their properties inside the nuclear medium. In particular, a reduction of the binding energies leads to their dissolution with increasing density and the Mott effect is observed. The description of light clusters has been developed already in some detail. For heavier clusters, the dependence of their binding energies has still to be extracted from microscopic calculations in the whole range of thermodynamical variables.

The formation and dissolution of clusters at low densities causes an increase of the nuclear symmetry energy at low densities. This feature can be studied experimentally in heavy-ion collisions and is relevant in astrophysical simulations. Providing extensive EoS tables is one important aim for the application of the gRDF approach in the future.

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